The Role of Autonomous Non-Capacity Creating Expenditures in Recent Kaleckian Growth Models: an Assessment from the Perspective of the Sraffian Supermultiplier Model

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Resumo

Uma literatura Kaleckiana recente, incorpora gastos autônomos que não criam capacidade em modelos de crescimento de médio e de longo prazos. Neste artigo nós analisamos esta literatura tendo como referência básica o modelo do Supermultiplicador Sraffiano. Deste último ponto de vista, os modelos Kaleckianos tradicionais de crescimento apresentam três limitações importantes: eles não conseguem gerar uma tendência a uma situação de utilização normal da capacidade; eles não preveem a existência de uma relação positiva entre a taxa de crescimento do produto e a taxa de investimento; e eles não admitem que os gastos que não criam capacidade tenham um papel determinante na explicação das trajetórias observadas de crescimento liderado pela demanda. Nós mostramos que o modelo de médio prazo não permite superar, de maneira completa, todas essas limitações e ainda pode gerar comportamentos implausíveis sob certas condições. Já o modelo de longo prazo permite superar as três limitações mencionadas porque combina uma função de investimento com base no princípio do ajustamento do estoque de capital com uma determinação endógena da taxa de poupança que não recorre a mudanças na distribuição de renda. Além disso, argumentamos que não é correto usar uma análise baseada no princípio da instabilidade Harrodiana para investigar a condição de estabilidade do equilíbrio do tipo Supermultiplicador do modelo de longo prazo. Em vez disso, sugerimos que a condição de estabilidade de tal equilíbrio deve ser interpretada como uma condição de estabilidade keynesiana generalizada segundo a qual a propensão a gastar é menor do que um ao longo de toda trajetória de desequilíbrio do modelo.

Abstract

A recent literature incorporates autonomous non-capacity creating expenditures in a medium-run and a long-run Kaleckian growth models. We analyze this literature from the perspective of the Sraffian Supermultiplier model. From this point of view, traditional Kaleckian growth models have three important limitations: they cannot generate a tendency towards normal capacity utilization; they do not imply a positive relationship between the rate of output growth and the investment-output ratio; and they fail to give a growth-determining role to autonomous non-capacity creating expenditures in the interpretation of real demand-led growth processes. The medium-run model cannot completely overcome all these three limitations and can generate implausible behavior under certain conditions. The long-run model can overcome all three limitations because it combines an investment function based on the capital stock adjustment principle with an endogenous determination of the saving ratio that does not resort to changes in income distribution. Moreover, we argue that an analysis based on the Harrodian instability principle cannot be properly used in the investigation of the stability condition of the Supermultiplier equilibrium of the long-run model. Instead, we suggest that the stability condition of such an equilibrium should be interpreted as a generalized Keynesian stability condition, according to which the propensity to spend should have a value lower than one throughout the whole disequilibrium path of the model.

Área 2: Distribuição de Renda e Crescimento Econômico

1. Introduction

The paper analyzes some of the findings of the recent literature on Kaleckian growth models that incorporates an autonomous non-capacity creating expenditure (hereafter NCCE) component from the perspective of the Sraffian Supermultiplier model (hereafter SM-model). From this viewpoint, traditional Kaleckian growth models have three important limitations: they cannot generate a tendency towards normal capacity utilization; they do not imply a positive relationship between the rate of output growth and the investment-output ratio; and they fail to give a growth-determining role to autonomous NCCEs in the interpretation of real demand-led growth processes. In order to deal with the first of these limitations, Allain (2015), Lavoie (2016) and Dutt (2016) proposed two Kaleckian growth models that incorporate an autonomous NCCE component in aggregate demand. The medium-run model maintains the traditional Kaleckian investment function that is not compatible with the capital stock adjustment principle, while the long-run model incorporates an investment function that is compatible with such a principle. We analyze these models to verify whether and to what extent they can overcome the limitations mentioned above. We also complement the existing analysis of these models where: we believe that they are incomplete; we can contribute to the clarification of their behavior and the economic meaning of the results obtained from them. In these analyses, we take the SM-model as our main reference.

The paper is organized as follows. Section 2 discusses a basic version of the traditional Kaleckian growth models and points out its main limitations from the point of view of the SM-model. We also analyze a modified version of the traditional Kaleckian model in which we introduce an investment function based on the capital stock adjustment. The medium-run model is presented and assessed in section 3. Section 4 examines the long-run model. Finally, in section 5 we present some concluding remarks.

2. Some limitations of traditional Kaleckian growth models

2.1. Traditional Kaleckian growth models

Let us first discuss a basic version of the traditional Kaleckian growth model for a closed capitalist economy without a government sector. In this model, the only method of production in use requires a fixed combination of a homogeneous labor input with homogeneous fixed capital to produce a single product. Natural resources are supposed to be abundant, constant returns to scale prevails, there is no technological progress, and economic growth is not constrained by labor scarcity.

Under these assumptions, the level of capacity output depends on the level of the capital stock K and on the technical capital to capacity output ratio v as:

$$(1)\quad Y_P = \frac{1}{v}K$$

where Y_P is the level of capacity output.

On the other hand, following the principle of effective demand, the model supposes that real aggregate demand determines the equilibrium level of real aggregate output.[1] Indeed, to further simplify the analysis of the model we assume that output adapts quickly to demand, because either "short-term expectations" are always realized or that they are quickly revised in light of recent experience. Hence, the goods market is always in equilibrium (i.e., the short-run equilibrium of the Kaleckian models).

Aggregate demand is composed of three types of expenditures: consumption out of wages, consumption out of profits and capitalist investment. Consumption is a function of income distribution, and total income is distributed as wages and profits. The distribution of income is determined along Kaleckian lines, with firms setting their real mark-up over costs and, therefore, the shares of profits and wages on total income. As a simplification, it is also supposed that the determination of income distribution is independent of the values of the endogenous variables of the model, so that income distribution is an exogenous variable. Consumption also depends on the consumption habits of those who receive wages and profits, which are represented by given parameters in traditional Kaleckian models. We will follow the usual practice of supposing that consumption out of wages is equal to the wage bill of the economy (i.e., the marginal propensity out of wages is equal to one) and that the marginal propensity to consume out of profits is positive and smaller than one.

With these assumptions, consumption out of wages (C_W) and profits (C_K) can be represented by the following equations:

(2) $C_W = (1 - \pi)Y$

and

(3) $C_K = (1 - s_K)\pi Y$

where π is the exogenous profit share of income and the corresponding wage share is equal to $1 - \pi$. The parameter s_K (with, $0 < s_K < 1$) is the marginal propensity to save out of profits and its complement, $1 - s_K$, is the marginal propensity to consume out of profits. Note that since both π and s_K are exogenous, then C_W and C_K are directly proportional to output. Therefore, total consumption is also proportional to output and the constant of proportionality is the marginal propensity to consume out of total income $c = 1 - \pi + (1 - s_K)\pi$, which determines the share of NCCEs in total aggregate demand under the specific assumptions of the model. The marginal propensity to consume is equal to the average propensity to consume since there is no autonomous consumption component in the traditional Kaleckian model.

Capitalist investment[2] depends on the existing stock of capital and the pace of capital accumulation (i.e., $I = g_K K$). The model assumes that capitalist firms are able to determine the pace of capital accumulation according to their desired rate of capital accumulation (i.e. $g_K = g_K^d$) given by:

(4) $g_K^d = \alpha + \beta(u - u_n)$

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[1] In fact, all variables in the model are measured in real terms. The same applies to the other models discussed here. Thus, in order to simplify the exposition, the term 'real' will be hereafter omitted.

[2] As is usual in the Kaleckian literature, the model measures investment, savings, profits and income in net (of depreciation) terms. We must say, however, that this procedure implies that the model cannot deal with negative rates of capital accumulation. Hence, one has to impose restrictions to the values of the parameters and exogenous variables of the model in order to guarantee that the rate of capital accumulation have only non-negative values.

where $u = Y/Y_P = vY/K$ is the actual rate of capacity utilization (with, $0 \leq u \leq 1$) and u_n is the normal rate of capacity utilization (with, $0 < u_n < 1$). The parameter β (with, $\beta > 0$) measures the sensitivity of g_K^d to the difference between actual and normal rates of capacity utilization, while the parameter α represents the trend rate of growth of sales according to firm's expectations, which is usually specified as a positive magnitude (i.e. $\alpha > 0$). Hence, the following equation gives us the investment function of the traditional Kaleckian model:

$$(5)\ \ I = \big(\alpha + \beta(u - u_n)\big)K$$

This is one of the specifications for the investment function adopted by Allain (2015), Lavoie (2015) and Dutt (2016). First, observe that the investment function above has an autonomous and an induced component. The former component can be represented as θK, where $\theta = \alpha - \beta u_n$, while the latter component can be expressed as $\beta uK = \beta vY$. Thus βv is the marginal propensity to invest of the economy. Secondly, it is important to note that, although such an investment function has an induced component, it is not compatible with the capital stock adjustment principle and, therefore, it does not belong to the family of investment functions based on the accelerator type of mechanism. According to this principle, inter-capitalist competition influences the process of investment leading to the tendency towards the adjustment of productive capacity to meet demand at a price that covers the production expenses and allows, at least, the obtainment of a minimum required profitability. From the perspective of the capital stock adjustment principle, capitalist competition induces firms to try to produce a tendency towards normal capacity utilization by means of their investment decisions.[3] In contrast, one can easily verify that the investment function of the traditional Kaleckian model cannot produce such a tendency. Indeed, from equation (4) we can see that any possible divergence between u and u_n can continue indefinitely without causing any change in the desired rate of capital accumulation.

Let us now turn our attention to the determination of the endogenous variables of the model. We start by determining the equilibrium value of the rate of capacity utilization in the model. Inserting equations (2), (3) and (5) into an equation representing the equilibrium condition for the goods market, we can obtain the equilibrium level of output

$$(6)\ \ Y = \left(\frac{1}{s_K\pi}\right)(\theta + \beta u)K = \left(\frac{1}{s_K\pi - \beta v}\right)\theta K$$

Since by definition we have $K = (v/u)Y$, we can determine the equilibrium rate of capacity utilization in the traditional Kaleckian model (u_{Trad}) as follows:

$$(7)\ \ u_{Trad} = \frac{v\theta}{s_K\pi - \beta v}$$

To ensure that u_{Trad} has a positive value one must assume that $\theta = \alpha - \beta u_n > 0$ and that $s_K\pi > \beta v$. The Kaleckian literature calls the latter condition, Keynesian stability condition. Indeed, since $s_K\pi = 1 - c$, the stability condition can be rewritten as $c + \beta v < 1$, which means that it is equivalent to the condition that the marginal propensity to spend (equal to the marginal propensity to consume plus the marginal propensity to invest) has a value lower than unity.

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[3] See Goodwin (1951) and Chenery (1952). See also Matthews (1959) for a more detailed account of the capital stock adjustment principle.

On the other hand, from equation (6), we know that in equilibrium the value of output is proportional to the value of the capital stock. Thus, in the short-run equilibrium of the traditional Kaleckian model, the rate of growth of output is equal to the equilibrium rate of capital accumulation (i.e., $g = g_K$). To find an expression for the equilibrium rate of growth of output g_{Trad} we can insert the expression on the right hand side of equation (7) into equation (4) obtaining:

$$(8)\ \ g_{Trad} = \alpha + \beta(u_{Trad} - u_n) = \frac{s_K \pi \theta}{s_K \pi - \beta v}$$

Observe that the same conditions used to ensure that u_{Trad} has a positive value are also sufficient to guarantee the existence of a positive solution for g_{Trad}.

Finally, as a preparation for the discussion below, we will also deal with the determination of the investment to output ratio in the traditional Kaleckian model. The Kaleckian literature does not usually discuss such topic. However, as we shall see this topic is very important because the ratio of investment to output is a measure of the share of capacity creating expenditure in total aggregate demand and, therefore, an expression of the division of aggregate demand between capacity and non-capacity creating expenditures. In fact, as the adjustment of the actual rate of capacity utilization towards its normal level requires changes in this share, we must deal with the determination of the investment-output ratio here. It follows that from equation (6) we can verify that $I/Y = (\theta + \beta u)K/Y = s_K \pi$ and, therefore, that:

$$(9)\ h_{Trad} = s_K \pi$$

where h_{Trad} represents the value of the investment to output ratio in the traditional Kaleckian model. Since consumption/savings habits and the profit share are given from outside the model, the marginal propensity to save $s_K \pi$ (equal to the saving ratio) determines the investment to output ratio. The latter result means that the exogenously given share of non-capacity creating expenditures in total aggregate demand determines the share of capacity creating expenditure. Therefore, in the traditional Kaleckian model the division of aggregate demand between capacity and non-capacity creating expenditures cannot change endogenously to allow the adjustment of the actual rate of capacity utilization towards its normal level.

2.2. Some limitations

From the perspective of the SM-model, traditional Kaleckian growth models, such as the one presented in the last section, have three important limitations:

L.1 - they cannot generate a tendency towards normal capacity utilization;

L.2 - they do not imply a positive relationship between the rate of growth of output and the investment-output ratio;

L.3 - they fail to give a growth-determining role to NCCEs in the interpretation of real demand-led growth processes.

The first limitation mentioned above is the most discussed one in the critical literature on traditional Kaleckian growth models. The criticism of these models has both a theoretical and an empirical dimension. Regarding the former dimension, the criticism is directed to the idea that the actual rate of capacity utilization may deviate *permanently* from the normal rate of capacity utilization desired by firms without any repercussion on their investment decisions.[4] We would like to add that in traditional Kaleckian growth models the latter lack of repercussion is also independent of the *size* of the (positive or negative) divergence between actual and normal rates of capacity utilization. On the other hand, from an empirical perspective, these models are criticized for the fact that they would not be compatible with the mean reversion behavior that characterizes observed time series of capacity utilization, which is viewed as evidence in favor of the existence of a tendency towards normal capacity utilization.[5]

In principle, one may think that we could overcome such limitation by replacing the traditional Kaleckian investment function by one based on the capital stock adjustment principle. However, as the Kaleckian literature shows, this is not the case (see, Hein, Lavoie and Treeck, 2011 and 2012). To show that, let us suppose that the parameter α of the Kaleckian investment function becomes a variable that changes according to the following differential equation:

$$(10)\ \alpha' = \lambda(u - u_n) = \lambda\left(\frac{v(\alpha - \beta u_n)}{s_K\pi - \beta v} - u_n\right)$$

where $\alpha' = d\alpha/dt$ is time derivative of α and λ is an exogenous and strictly positive parameter (i.e. $\lambda > 0$). The parameter λ measures the sensitivity of α to the difference between actual and normal rates of capacity of utilization. Equation (10) represents the operation of the capital stock adjustment principle according to which whenever u is above (below) u_n firms try to adjust productive capacity to demand by increasing (decreasing) the pace of capital accumulation.

With this new specification of the investment function, the equilibrium values of the rate of capacity utilization, the rate of growth of output and the investment to output ratio would be:[6]

$$(11)\ u_{Har} = u_n$$

$$(12)\ \alpha_{Har} = g_{Har} = \frac{s_K\pi u_n}{v}$$

$$(13)\ h_{Har} = s_K\pi$$

where we use the subscript "*Har*" to denote that the equilibrium values are similar to the ones obtained in Harrod's growth model (Harrod, 1939 and 1948). The first remark to be made is that the modification introduced in the traditional Kaleckian model transforms it from a demand (investment) led growth into a supply constrained model of economic growth. In fact, the equilibrium rate of growth is essentially the so-called warranted rate of growth suggested originally by Harrod (1939) and shares with the latter rate of growth the feature that only supply side variables/parameters enter in its

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[4] On this type of criticism, see Committieri (1986), Auerbach and Skott (1988), Skott (2012) and Cesaratto (2015), among others.

[5] See, for instance, Skott (2012), DeJuan (2013) and Duménil and Lévy (2014) on this type of criticism. On the other hand, see, for instance, Corrado and Mattey (1997) and Schoder (2014) for the empirical evidence on the mean reversion behavior of the time series of capacity utilization.

[6] We have an equilibrium when $\alpha' = 0$. Hence, as $\lambda > 0$, imposing the latter condition in equation (10) implies that $u = u_n$, which gives us equation (11). From the condition $u = u_n$ in equation (4), we obtain $g = g_K = g_K^d = \alpha$. Next, introducing the latter result in equation (6) and solving for g we obtain the expression in equation (12). Finally, as already shown before, the equilibrium value of h follows directly from equation (6).

determination.[7] On the other hand, none of the parameters of the investment function, besides u_n, appear in equation (12).

The particular model under analysis also shares with the Harrodian growth model the result related to the "fundamental instability" of the equilibrium rate of growth. Indeed, according to equation (10), whenever the actual rate of capacity utilization is above (below) its normal level firms would try to bring the former in line with the latter by increasing (decreasing) the rate of capital accumulation. Nevertheless, they would not succeed in doing so. Actually, in trying to adjust the actual rate of capacity utilization towards its normal level, firms end up pushing the economy further away from it. The increase (fall) in α instead of causing a reduction (an increase) in the actual rate of capacity utilization provokes an increase (decrease) in it. More formally, assuming that $s_K\pi - \beta v > 0$), from equation (10) we have:

$$(14)\quad \frac{\partial \alpha'}{\partial \alpha} = \frac{v\lambda}{s_K\pi - \beta v} > 0$$

since $v, \lambda > 0$. The result shows that the equilibrium with normal capacity utilization is unstable in a strong sense, meaning that the instability is independent of the intensity of the adjustment process as measured by the parameter λ. It depends only on the positive sign of such parameter, which is specified in accordance with the capital stock adjustment principle.

Based on the previous analysis, it might be tempting to blame the investment function based on the capital stock adjustment principle for the instability result. Nonetheless, in our opinion, doing so would be a mistake. In reality, the instability result follows from the *combination* of such an investment function with the basic assumptions of the traditional Kaleckian model maintained in the modified model. More specifically, the instability result is due to the combination of the new investment function introduced in the model and the determination of the investment to output ratio by an exogenous saving ratio (i.e., equation (13) above). Given the capital to capacity output ratio, the process of adjustment of capacity to demand at a normal rate of utilization requires the variability of the investment to output ratio, that is the variability of the share of capacity creating expenditures on output. Hence, along the traverse from a low (high) rate of growth steady-state with normal capacity utilization towards a high (low) rate of growth steady-state also characterized by normal capacity utilization, the investment to output ratio must be able to increase (decrease) by the necessary amount to bring about the equality between actual and normal rates of capacity utilization.[8]

To understand the latter result more clearly, let us consider the following expression that represents the condition according to which a sustainable process of economic growth requires the reconciliation between the rates of growth of demand/output and productive capacity:

$$(15)\quad g = g_K = \frac{h}{v}u$$

Thus, given v and assuming that in the initial and final steady-states we have $u = u_n$, if the condition expressed in equation (15) is to be satisfied, a rise (fall) in the steady-state rate of growth of

[7] Since the marginal propensity to invest $h_{Har} = s_K\pi$ is equal to the complement of the marginal propensity to consume, then the marginal propensity to spend must be equal to unity. (i.e. $c + s_K\pi = 1 - s_K\pi + s_K\pi = 1$). This condition is usually associated with the validity of the Say's Law. In passing, note also that, in equilibrium, βv is no longer the marginal propensity to invest in the model, which is now equal to $s_K\pi$.

[8] To be sure, such argument does not require the assumption that actual and normal rates of capacity utilization remain equal to each other along the entire traverse path from one steady-state to the other.

demand/output g must be accompanied by an appropriate increase (reduction) in the investment to output ratio h. However, this cannot happen, both in the traditional and in the modified model. For they suppose that $h = h_{Trad} = h_{Har} = s_K\pi$ and v are given. Therefore, the only way to promote the reconciliation between g and g_K after an increase (decrease) in g is by means of a change in u in the same direction and by the appropriate magnitude. In traditional Kaleckian models, this type of modifications in g leads to stable changes in the endogenously determined equilibrium rate of capacity utilization.[9] On the other hand, in the modified model the same type of modifications leads to instability because whenever u is above (below) u_n firms cannot bring about a reduction (an increase) in u by increasing (reducing) the pace of capital accumulation since the ratio of investment to output h cannot rise (fall) appropriately.

The investment to output ratio cannot change because in these models the exogenous saving ratio determines its value. The basic assumptions of these models relating to the determination of income distribution (i.e. of the profit share π) and of the marginal propensity to consume out of profits (as implicitly represented by the parameter s_K) imply the exogeneity of the saving ratio. Therefore, overcoming the first limitation of traditional Kaleckian growth models requires changes in the basic assumptions of Kaleckian models in order to make the saving ratio and, hence, the share of capacity creating expenditures on output endogenous variables.

The second limitation is not usually debated in the Kaleckian literature, although it is closely related to the first one, as we suggested right above. In addition to the problems already discussed, we want to emphasize that the inability of traditional Kaleckian models in obtaining a positive relationship between the rate of growth of output and the investment to output ratio is a problem in itself. Such is the case because there seems to be some robust empirical evidence showing that this relationship can be identified in available economic databases. In this connection, see for instance the evidence found in the literature on growth empirics such as Lipsey and Kravis (1987), De Long and Summers (1991 and 1992), Blomström, Lipsey and Zejan (1996) and Sala-i-Martin (1997). Moreover, the investment function based on the flexible accelerator mechanism usually implies this kind of relationship. As since this latter type of investment function has a good record in empirical analysis, the latter provides further (although indirect) evidence for the existence of the empirical relationship that we are discussing. In this latter respect, see among others Chirinko (1993) and Chirinko, Fazzari and Meyer (1999 and 2011).

Finally, the third limitation of traditional Kaleckian growth models pointed out above refers to the fact that such models fail to give a growth-determining role for NCCE in their analysis of real processes of economic growth. In fact, traditional Kaleckian models are investment-led growth models. The introduction of other NCCEs in addition to the consumption out of profits and wages (e.g., government consumption and exports) in these models does not change this feature. They are usually specified as fixed proportions of the capital stock or the level output. Thus, the rate of capital accumulation determines the rate of expansion of these expenditures. Exogenous changes in the ratio of these expenditures to capital stock (or output) only affect the rate of growth of output by means of its effects on the equilibrium rate of capacity utilization. An increase (a decrease) in these ratios raises (reduces) the equilibrium rate of capacity utilization, which provokes an increase (a decrease) in the pace of capital accumulation and output growth. Therefore, NCCEs do not have an independent role in the determination of the pace of expansion of the economy in traditional Kaleckian growth models.

[9] Provided the Keynesian stability condition prevails.

Nevertheless, there is some evidence that the evolution of output largely explains the evolution of capitalist investment as can be found, for instance, in Chirinko (1993) Chirinko, Fazzari and Meyer (1999), Blomström, Lipsey and Zejan (1996), and Schoder (2014). Moreover, there is also some evidence that there exists a diversity of patterns of demand-led economic growth. Some countries seem to be driven by domestic NCCEs such as durable consumption, residential investment, and government spending, while other countries seem to be driven by the evolution of exports (i.e., a foreign NCCE). In this connection, refer to McCombie and Thirlwall (1994), Wen (2007), Leamer (2007 and 2009), Fiebiger (2014) and Girardi and Pariboni (2016), among others. This type of evidence challenges the basic neo-Kaleckian view of economic growth as essentially an investment driven process. Clearly, this is an important limitation of the traditional Kaleckian growth models when applied to the analysis of observed experiences of demand-led growth.

The controversy generated around the first limitation[10] engendered two types of reaction from the part of Kaleckian authors. The first type of reaction is characterized by a very defensive attitude towards traditional Kaleckian growth models. As summarized by Hein, Lavoie and Treeck (2012), some authors argue that the concept of normal capacity utilization and the corresponding long-run analysis are not relevant for Kaleckian growth analysis. Other authors believe that the existence of alternative and incompatible targets influencing firm's decisions may prevent the convergence towards the normal (target) rate of capacity utilization. Finally, another group of Kaleckians argues that the latter rate may be treated as an endogenous variable in the long-run, because of the influence of a hysteresis mechanism in the definition of the normal rate.[11]

On the other hand, the second type of reaction involves authors who accept to introduce changes in some of the basic assumptions of traditional Kaleckian models. Allain (2015), Lavoie (2016) and Dutt (2016) introduce both an investment function based on the capital stock adjustment principle and an autonomous NCCE in their models. As we shall see below, these changes allow them to overcome all tree limitations. They start their analysis by developing a medium-run Kaleckian model that combines the traditional Kaleckian investment function with an autonomously growing NCCE. Finally, in the long-run model, they introduce the investment function based on the capital stock adjustment principle while maintaining the autonomous NCCE component. In what follows, we will discuss their contributions in more detail, as well as their relation to the SM-model literature.

3. The medium-run Kaleckian growth model

3.1. The model

Following Lavoie (2016), the only modification in the medium-run model, in comparison to the traditional Kaleckian model, is the inclusion of an autonomous component in the consumption function out of profits. Let us suppose that Z is this autonomous NCCE and that it grows at an exogenous rate g_Z. In this case, we must replace equation (3) by:

[10] The existing literature has not appropriately addressed the other two limitations.

[11] For a detailed account of these arguments and related references, see Hein, Lavoie and Treeck (2012).

(16) $C_K = (1 - s_K)\pi Y + Z$

The medium-run model preserves all other assumptions and equations of the traditional Kaleckian growth model. In particular, it maintains the Kaleckian investment function as represented by equation (5).

Now, supposing that the goods market is in equilibrium and using equations (2), (16) and (5), we may obtain an expression for the level of output:

(17) $Y = \left(\frac{1}{s_K\pi}\right)[(\theta + \beta u)K + Z] = \left(\frac{1}{s_K\pi - \beta v}\right)(\theta K + Z) = \left(\frac{1}{s_K\pi - \beta v}\right)(\theta + z)K$

where $z = Z/K$. Note that we now have two autonomous aggregate demand components: autonomous investment θK and autonomous consumption Z. The former is an autonomous capacity creating expenditure and the latter an autonomous NCCE. Moreover, since g_Z is an exogenous variable, it may be different from the rate of capital accumulation g_K. Thus, the ratio z changes according to the following differential equation:

(18) $z' = z(g_Z - g_K)$

where $z' = dz/dt$ is the time derivative of z.

Allain (2015), Lavoie (2016) and Dutt (2016) define the short-run equilibrium for this model as a situation in which all endogenous variables, except z, are in equilibrium conditional on a given value of z. Hence, following a similar procedure to the one used in the derivation of the equilibrium values of u, g and h in the traditional Kaleckian model we obtain:

(19) $u = \frac{v(\theta + z)}{s_K\pi - \beta v} = u_{Trad} + \frac{vz}{s_K\pi - \beta v}$

(20) $g_K = \frac{s_K\pi\theta + \beta vz}{s_K\pi - \beta v} = g_{Trad} + \frac{\beta vz}{s_K\pi - \beta v}$

(21) $h = \frac{s_K\pi\theta + \beta vz}{\theta + z} = \frac{h_{Trad}\theta + \beta vz}{\theta + z}$

where we use the symbols u_{Trad}, g_{Trad} and h_{Trad} to draw attention to the fact that the equilibrium values of the traditional Kaleckian model are included within the solution of medium-run model.

From equations (19), (20) and (21), we can verify that, as long as the value of z changes, the short-run equilibrium values of the endogenous variables also change. Thus, the medium-run equilibrium of the model under analysis occurs when the ratio z achieves a stationary value. According to equation (18), we obtain a stationary value of z if $z' = 0$. The latter condition can be satisfied if either $z = 0$ or $z > 0$ and $g_K = g_Z$. Therefore, there are two possible steady states in the medium-run Kaleckian model.[12] Let us analyze each of them.

[12] Lavoie (2016) and Dutt (2016) do not analyze the equilibrium with $z = 0$, because they conduct their dynamic analysis of the model in terms of proportional rates of changes (see Skott (2017) for a similar objection). On the other hand, Allain (2015) admits the existence of two equilibria in the medium-run model, but fails to interpret the economic meaning and the implications of the equilibrium with $z = 0$.

With $z = 0$ we have the same equilibrium values as in the traditional Kaleckian growth model as follows:

$$(22)\ \ z_{Trad} = 0$$

$$(7)\ \ u_{Trad} = \frac{v\theta}{s_K\pi - \beta v}$$

$$(8)\ \ g_{Trad} = \frac{s_K\pi\theta}{s_K\pi - \beta v}$$

$$(9)\ \ h_{Trad} = s_K\pi$$

Here it is important to be remarked that the equilibrium with $z = 0$ has an economic meaning and, therefore, it should not be ignored in the analysis of the medium-run Kaleckian model.

On the other hand, with $z > 0$ we must have $g_K = g_Z$ if $z' = 0$ is to be met. Thus, from (20) we have:

$$g_{Trad} + \frac{\beta v z}{s_K\pi - \beta v} = g_Z$$

Solving the equation above for z we may obtain the equilibrium value of the ratio z as follows:

$$(23)\ z_{hyb} = \frac{(s_K\pi - \beta v)}{\beta v}(g_Z - g_{Trad})$$

By the definitions of the variables involved, the ratio z cannot assume negative values. Hence, supposing that the Keynesian stability condition is met (i.e., $s_K\pi - \beta v > 0$), the existence of this specific equilibrium with $z > 0$ requires the condition $g_Z > g_{Trad}$. Now, inserting z_{Hyb} into equations (19), (20) and (21) we obtain

$$(24)\ \ u_{hyb} = \frac{g_Z - \theta}{\beta} = u_n + \frac{g_Z - \alpha}{\beta}$$

$$(25)\ \ g_{hyb} = g_Z$$

$$(26)\ \ h_{hyb} = \frac{\beta v g_Z}{g_Z - \theta}$$

Where we use the subscript "*hyb*" to draw attention to the hybrid nature of the equilibrium since it shares characteristics of both, the traditional Kaleckian models and the long-run (Supermultiplier) model.

We must make three remarks on the second equilibrium of the medium-run Kaleckian model. First, from equation (25) we can see that the medium-run model can generate a pattern of economic growth in which NCCEs have a growth-determining role. Nevertheless, note that the latter result follows only when the second equilibrium prevails. Secondly, observe that the existence of an autonomous NCCE cannot alone produce a tendency towards normal capacity utilization as can be seen in equation (24). Such a tendency requires that investment expenditures cease to be a source of autonomous demand, allowing the adjustment of α towards g_Z. Finally, the hybrid equilibrium implies the existence of a *negative* relation between the investment to output ratio and the equilibrium rate of output. In fact, if $\theta > 0$ then from equation (26) we can obtain:

$$(27)\ \frac{\partial h_{hyb}}{\partial g_z} = -\frac{\theta\beta v}{[g_Z - \theta]^2} < 0.$$

The reason is the following. Consider, for instance, an increase in g_Z (and, therefore, in the rate of growth of output). Such an increase raises the ratio z and causes a reduction in h. This latter result obtains because a higher rate of growth in autonomous consumption induces more investment due to the component βu in the desired accumulation rate equation, but the existence of an autonomous investment component, θK (invariant to the growth in autonomous consumption), necessarily implies that the rate of capital accumulation initially falls behind. Thus, when there is an increase in the rate of growth of autonomous consumption we have, initially, $g_Z > g > g_I > g_K$. Only after sometime the rate of capacity utilization will have risen sufficiently to induce capital stock and aggregate investment to grow *pari passu* with output but with a permanently higher ratio z.

Now let us analyze the stability conditions of the two equilibria of the medium-run model. Thus, inserting the expression in (20) into equation (18) and taking the derivative of z' with respect to z, we obtain:

$$(28)\ \frac{\partial z'}{\partial z} = g_Z - g_{Trad} - \frac{2\beta vz}{s_K\pi - \beta v}$$

Next, let us evaluate the derivative above at each equilibrium. Doing so we obtain the following results:

$$(29)\ \left.\frac{\partial z'}{\partial z}\right|_{z=0} = g_Z - g_{Trad}$$

and

$$(30)\ \left.\frac{\partial z'}{\partial z}\right|_{z=z_{hyb}} = g_{Trad} - g_Z.$$

As can be verified, the steady state with $z = 0$ is stable if $g_{Trad} > g_Z$. This means that autonomous capital accumulation is relatively high, so that investment and induced consumption grow faster than Z. Hence, the influence of autonomous NCCE over total aggregate demand gradually diminishes and in the end the ratio z turns out to be negligible and the economy follows an investment-led growth path. Moreover, it is worth noting the implausible behavior of the model during the convergence towards the traditional steady state. Since, initially, we must have $z > 0$, then, as can be seen from equation (20), we have $g_K > g_{Trad} > g_Z$. As long as z tends asymptotically towards zero, g_K will tend asymptotically to g_{Trad}, falling during the traverse. Since the rate of growth of output growth is a weighted average of g_K (rate of growth of autonomous investment) and g_Z, it also will fall during the traverse. But the investment-output ratio will increase throughout the traverse, asymptotically tending towards $s_K\pi$, the maximum value of the investment-output ratio.[13] So this particular steady state not only presents an investment-output ratio which is invariant to the growth rate, but it also presents an out-of-equilibrium investment-output ratio continuously increasing as the economy is slowing down. On the other hand, the equilibrium with $z > 0$ is stable if $g_Z > g_{Trad}$. In

[13] The investment-output ratio can be written as $h = (Y - C)/Y = s_K\pi - Z/Y = s_K\pi - vz/u$. So $z > 0$ implies $h < s_K\pi$.

this case, Z grows relatively fast, inducing more and more investment, thus gradually making autonomous investment θK less important than induced investment βvY, and in the end the economy follows a pattern of economic growth led by NCCEs.

3.2. An assessment of the medium-run model

We may now discuss the results of our analysis of the medium-run model. Let us start by saying that if we think of such a model as an intermediate step towards the long-run model, then we believe that there is no problem in using it. The problem occurs when we think of the medium-run model as a contribution to the understanding of economic reality in itself, independent from the long-run model.[14] Let us see why.

First, the medium-run does not allow us to overcome all the limitations of the traditional Kaleckian models discussed in the previous section. The model cannot generate a tendency towards normal capacity utilization. To obtain such a result, it would be necessary that $\alpha = g_Z > g_{Trad}$,[15] which is of course a very restrictive condition since all variables involved are exogenous. As we will see shortly, the long-run model eliminates this obstacle by making α an endogenous variable that in equilibrium is equal to g_Z and, at the same time, takes out of the scene completely the traditional Kaleckian equilibrium. On the other hand, the medium-run model does not predict the existence of a positive relationship between the investment-output ratio and the rate of growth of output. In the case of the equilibrium of the traditional-type, there is no relation between these two variables, while in the other equilibrium the model predicts a negative relationship between the two variables. Hence, it cannot overcome the second limitation. Nonetheless, observe that the medium-run model can overcome the third limitation, although only partially. In the equilibrium of the traditional-type NCCEs have no role in the growth process. In fact, as we saw, such an equilibrium is characterized by the zero value of the ratio z. Only when the other equilibrium prevails (i.e., when we have $g_Z > g_{Trad}$) NCCEs have a leading role in the determination of the trend rate of growth of output.

Secondly, the medium-run model, as an autonomous entity, is not a plausible representation of reality. In this respect, the main difficulty is that the model has a problem of *double identity*. As we argued, a complete analysis of the model reveals that it has two alternative equilibria. When one of them is stable, the other is not. In principle, there is no restriction on the values of the exogenous variables and parameters of the model that allows us to rule out one of the two equilibria. In particular, there is no reason to expect that the situation in which we have $g_Z > g_{Trad}$ should prevail over a situation in which $g_Z < g_{Trad}$. The double identity problem of the medium-run model is evident when one recognizes that in one equilibrium we have an investment-led pattern of economic growth, while in the other the economy is led by a NCCE. The convergence towards the investment-led equilibrium requires that the influence of the autonomous NCCE diminishes until it becomes nil. Now imagine that, instead of a closed economy without government, we were dealing with a model of an open economy with government activity, in which exports and government expenditures would play the role

[14] For instance, Dutt (2016) attributes to the medium-run model the status of an alternative closure for heterodox growth theory. We believe that only the long-run (Supermultiplier) model should deserve such a status.
[15] Note that this result requires that $\alpha = g_Z < g_{Har} = s_K \pi u_n / v$. This is what Dutt (2016) calls "rational expectation" Supermultiplier model.

of autonomous NCCEs. Does a situation in which such NCCEs completely lose their influence over the rate of growth of output make any sense? Moreover, note that while the economy converges to the investment-led equilibrium, with z asymptotically diminishing towards zero, the investment to output ratio continuously increases towards its maximum value given by $h_{Trad} = s_K\pi$. Such behavior of the investment to output ratio is not plausible.

It seems to us that the cause of the problem of double identity of the medium-run model is related to the presence, within the same model, of two sources of autonomous aggregate demand, one related to a capacity creating expenditure (i.e. autonomous investment, θK) and the other related to a NCCE (i.e. autonomous consumption, Z). This feature of the model can be clearly identified when we compare equations (6) and (17). To justify our claim let us show what happens if we completely take out the autonomous investment component from the medium-run model.

To do so, we must suppose that $\theta = 0$. From equation (18) we can verify that the model still generates two equilibria as before. However, now the equilibrium associated with a zero value of the ratio z has different characteristics. First, since $\theta = z = 0$, from equations (19), (20) and (21) we can see that the medium-run equilibrium values are:

$$(19') \; u = \frac{v(\theta + z)}{s_K\pi - \beta v} = u_{Trad} + \frac{vz}{s_K\pi - \beta v} = 0$$

$$(20') \; g_K = \frac{s_K\pi\theta + \beta vz}{s_K\pi - \beta v} = g_{Trad} + \frac{\beta vz}{s_K\pi - \beta v} = 0$$

$$(21') \; h = \frac{s_K\pi\theta + \beta vz}{\theta + z} = \frac{h_{Trad}\theta + \beta vz}{\theta + z} = 0$$

The equilibrium is no longer the same as in the traditional Kaleckian model. Moreover, from equation (29)

$$(29') \; \left.\frac{\partial z'}{\partial z}\right|_{z=0} = g_Z > 0$$

since by definition g_Z has a positive value. Hence, the equilibrium with $z = 0$ is unstable now.

The medium-run equilibrium with $z > 0$ also changes.[16] Making $\theta = 0$ in equations (23), (24), (25) and (26) we obtain:

$$(23') \; z_{hyb} = \frac{(s_K\pi - \beta v)}{\beta v} g_Z$$

$$(24') \; u_{hyb} = \frac{g_Z}{\beta} = u_n \frac{\left(\frac{v}{u_n}\right) g_Z}{\beta v}$$

$$(25') \; g_{hyb} = g_Z$$

$$(26') \; h_{hyb} = \beta v$$

[16] We would like to point out that the modified medium-run model discussed here is very similar to the SM-model with incomplete adjustment presented by Freitas and Serrano (2015).

From equation (25'), note that the economy is driven by the autonomous NCCE as in the original model. However, now the equilibrium value of the investment to output ratio is no longer negatively related to the rate of growth of output. It is determined by the marginal propensity to invest, which is constant in the medium-run model. The equilibrium rate of capacity utilization is still different from the normal rate and can only be brought into equality with latter rate if somehow β turns out to be an adjustment variable in the long-run in such a way that we have $\beta^{*}v = (v/u_n)g_Z$.[17] More importantly, the stability analysis of the equilibrium reveals that:

$$(30')\ \left.\frac{\partial z'}{\partial z}\right|_{z=z_{hyb}} = -g_Z < 0.$$

Therefore, the equilibrium is unambiguously stable. And, since the other equilibrium is now unstable, we have completely eliminated the problem of double identity of the model. To confirm, compare equation (17) of the original medium-run model to the equation below

$$(17')\ \ Y = \left(\frac{1}{s_K\pi - \beta v}\right)Z$$

Now it is clear that we have only one source of autonomous demand, the one related to the NCCE. In this connection, it is important to note that in contrast to the original medium-run model, the modified model can completely overcome the limitation of the traditional Kaleckian related to the growth-determining role of NCCEs in the process of economic growth.

4. The long-run Kaleckian growth model

In the previous section, we showed that the medium-run model fails in overcoming the limitations of the traditional Kaleckian model associated with the convergence towards normal capacity utilization and the positive relationship between investment-output ratio and the rate of growth of output. Here we will discuss the long-run model and show why it succeeds where the medium-run model fails. The most important difference between the medium and long-run models is that the latter discards the traditional Kaleckian investment function and replaces it by an investment function based on the capital adjustment principle (called by some authors as a Harrodian mechanism). In fact, as we will see, the success of the long-run model in overcoming all three limitations here discussed is due to the *combination* of the latter investment function with the presence of an autonomous NCCE in the model.

4.1. The model

The long-run model is comprised of the same equations as the medium-run except by the introduction of a fundamental change in the investment function. Such a change involves the endogenization of the autonomous component of the Kaleckian investment function in a way that is

[17] This would be the case if one supposes that $\beta' = \gamma(u - u_n)$.

compatible with the capital stock adjustment principle. We will use the same specification adopted in the discussion of the Harrodian extension of the traditional Kaleckian model:

(10) $\alpha' = \lambda(u - u_n)$.

We may now combine equations (18) and (10) to form the dynamical system that will be used in the analysis of the long-run model as follow:

$$(31)\begin{cases} z' = z(g_Z - g_K) \\ \alpha' = \lambda(u - u_n) \end{cases} = \begin{cases} z' = z\left(g_Z - \alpha - \beta\left[\dfrac{v(z+\alpha-\beta u_n)}{s_K\pi - \beta v} - u_n\right]\right) \\ \alpha' = \lambda\left[\dfrac{v(z+\alpha-\beta u_n)}{s_K\pi - \beta v} - u_n\right] \end{cases}$$

Let us start by analyzing the values of the endogenous variables in equilibrium. The latter occurs when we have $\alpha' = z' = 0$. Imposing this equilibrium condition to the system above we can see that $\lambda(u - u_n) = 0$ requires that $u = u_n$ since $\lambda > 0$. We can also verify that equation $z(g_Z - g_K) = 0$ has two possible solutions: $z = 0$ and $z > 0$, with $g_Z = g_K$. Thus, as in the case of the medium-run model, we have two equilibria, in which one of them is characterized by a value of the ratio z equal to zero. However, the equilibrium with $z = 0$ is *not* equivalent to the one in the traditional Kaleckian model. It is, in fact, equivalent to the equilibrium of the Harrodian extension of the traditional Kaleckian model as can be seen in equations (11), (12) and (13).[18] The equilibrium values of the endogenous variables are the following.

(32) $z_{Har} = 0$

(11′) $u_{Har} = u_n$

(12′) $\alpha_{Har} = g_{Har} = \dfrac{s_K\pi u_n}{v}$

(13′) $h_{Har} = s_K\pi$

We have normal capacity utilization, the rate of growth of output is equal to Harrod's warranted growth rate, and the investment-output ratio is determined by the marginal propensity to save. Thus, let us refer to this equilibrium as the Harrodian equilibrium of the long-run model.

We may now analyze the local stability conditions in the neighborhood of the Harrodian equilibrium. The Jacobian of the system (31) evaluated at this specific equilibrium point is given by:

$$(33)\ \boldsymbol{J_{har}} = \begin{pmatrix} g_Z - \dfrac{s_K\pi u_n}{v} & 0 \\ \dfrac{\lambda}{s_K\pi - \beta v} & \dfrac{\lambda}{s_K\pi - \beta v} \end{pmatrix}$$

For the stability of the equilibrium, the Jacobian must have a positive determinant and a negative trace. It can be shown that if $s_K\pi > \beta v$ holds it is not possible to have at the same time a negative trace and a positive determinant and, therefore, the Harrodian equilibrium is *unstable*.[19] Moreover, we may note

[18] The procedure used to find the solutions for u, α (or g) and h are similar to the one described in footnote 6.

[19] Thus, if $s_K\pi - \beta v > 0$ then the term $\lambda/(s_K\pi - \beta v)$ has a positive value, since λ is positive. Hence, the trace can only be negative if $g_Z - (s_K\pi u_n/v) < 0$. However, if the latter inequality holds the determinant has a negative value.

that the instability of such equilibrium does not depend on the specific value of the adjustment parameter λ, but only on its sign. Thus, we have instability in a strong sense.

Now, let us turn our attention to the other equilibrium of the long-run model, the one with a positive value of the ratio z. In this case it can be shown[20] that the endogenous variables have the following values in equilibrium:

$$(34)\ z_{sup} = \frac{s_K \pi u_n}{v} - g_Z = g_{Har} - g_Z$$

$$(35)\ u_{sup} = u_n$$

$$(36)\ \alpha_{sup} = g_{sup} = g_Z$$

$$(37)\ h_{sup} = \frac{v}{u_n} g_Z$$

Here we have normal capacity utilization in equilibrium, the process of economic growth is led by NCCE, and there is a positive relationship between the investment to output ratio and rate of growth of output. Since these results are similar to those obtained in the SM-model literature, we will refer to this equilibrium as the Supermultiplier equilibrium, which justifies the use of the subscript "sup". As can be verified, the Supermultiplier equilibrium of the long-run Kaleckian model allows us in principle to overcome all the limitations of the traditional Kaleckian growth model.

What remains to be shown is under what conditions such an equilibrium is stable. Hence, let us investigate the local stability conditions of the system (31) in the neighborhood of the Supermultiplier equilibrium. The Jacobian of the system evaluated at such equilibrium point is the following:

$$(38)\ \boldsymbol{J_{sup}} = \begin{pmatrix} \left(g_Z - \frac{s_K \pi u_n}{v}\right)\left(\frac{\beta v}{s_K \pi - \beta v}\right) & \left(g_Z - \frac{s_K \pi u_n}{v}\right)\left(\frac{s_K \pi}{s_K \pi - \beta v}\right) \\ \frac{\lambda}{s_K \pi - \beta v} & \frac{\lambda}{s_K \pi - \beta v} \end{pmatrix}$$

Here it can be shown that the condition for the determinant of $\boldsymbol{J_{sup}}$ to be positive is the following:

$$(39)\ c + h_{sup} < 1$$

where $c = 1 - s_K \pi$ is the marginal propensity to consume and $h_{sup} = (v/u_n) g_Z$ is the marginal propensity to invest in equilibrium. Thus condition (39) requires that the marginal propensity to spend in equilibrium must have a value lower than one in order to ensure that $Det\boldsymbol{J_{sup}}>0$**.** Observe that such condition does not depend on the intensity of the adjustment of capacity to demand as captured by the parameter λ. However, this condition is not sufficient to ensure the stability of the Supermultiplier equilibrium. Indeed, in order for the trace of $\boldsymbol{J_{sup}}$ to be negative the following condition must hold:

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[20] Thus, if $z' = 0$ and $z > 0$ then, from the first equation of the system (31), we obtain the equilibrium value of z. On the other hand, since $\lambda > 0$, if $\alpha' = 0$ then, from the second equation of the system (31), we have $u = u_n$, which is the equilibrium value of the rate of capacity utilization. Next, using the latter fact on the first equation of the system with if $z' = 0$ and $z > 0$ we obtain the equilibrium values of the rates of growth of output and capital as $\alpha = g = g_Z$. Finally, using the latter in the investment function we have $I = g_Z K$ and, therefore, $h = I/Y = (v/u_n) g_Z$.

$$(40)\ c + h_{sup} + \frac{\lambda}{\beta u_n} < 1$$

It is clear that once (40) is met then (39) also holds. Hence, condition (40) is the least restrictive sufficient condition for a positive determinant and a negative trace and, therefore, it is the stability condition for the Supermultiplier equilibrium of the long-run Kaleckian model. Observe that this stability condition depends not only on the sign of the adjustment parameter λ, but also on the magnitude of such a parameter. For sufficiently low values of λ, which represents a situation where firms try to gradually adjust productive capacity to demand, the Supermultiplier equilibrium is stable. That is, the stability of the equilibrium depends on the intensity of the adjustment process.

4.2. An assessment of the long-run model

First, note that none of the Kaleckian authors involved in the development of the long-run model analyzed its Harrodian equilibrium. However, contrary to what happened in the case of the analysis of the medium-run model, this omission does not have any serious consequence. In fact, since the Harrodian equilibrium is unstable in a strong sense, it cannot exert any influence over the endogenous variables of the long-run model and, therefore, it can be safely ignored.

Secondly, in contrast to the medium-run model, provided the stability condition of the Supermultiplier equilibrium is met, the long-run model can overcome all three limitations of the traditional Kaleckian model. In this case, the rate of capacity utilization converges towards its normal level, the investment-output ratio positively depends on the trend rate of growth of the economy and the expansion of autonomous NCCEs explains the trend rate of growth of output.[21] Here, it is important to emphasize that the success of the long-run model in overcoming the referred limitations follows from the *combination*, within the model, of an autonomous NCCE component in aggregate demand and an investment function based on the capital stock adjustment principle. In fact, as we saw above in the analysis of the modified Kaleckian (Harrodian) model, the introduction of the capital stock adjustment principle in a model with an exogenously given aggregate demand share of NCCE leads to an instability result of the Harrodian type. On the other hand, our analysis of the medium-run model showed that the introduction of an autonomous NCCE in a model without an investment function based on the capital stock adjustment principle partially fails to overcome all three limitations of the traditional Kaleckian model. Therefore, the success of the long-run model in overcoming these limitations should be attributed to the use of the principle of capital stock adjustment in the explanation of investment decisions *combined* with the existence of an autonomous NCCE within the model.

We believe that the role of autonomous NCCEs in the Kaleckian formulation of the long-run model is not sufficiently clear. Thus, we shall discuss this point in more detail by making use of elements of the analysis of the SM-model.[22] Indeed, besides being the long-run driver of the demand-led growth process, autonomous NCCE also allows the endogenous determination of the saving ratio (i.e., the average propensity to save) in the long-run model, even when one considers income distribution and consumption habits given from outside the model. The endogeneity of the saving ratio

[21] See equations (34)-(37) above in the text.

[22] See Serrano (1995a, 1995b), Freitas & Serrano (2015) and Serrano & Freitas (2017).

allows the movements of the investment-output ratio that are necessary for bringing the actual rate of capacity utilization in line with its normal level.

To understand why this happens, we can express the saving ratio (using our notation here) as follows:

$$(41)\ \left(\frac{S}{Y}\right)_{sup} = s_K\pi - \frac{v}{u_n} z_{sup} = s_K\pi - \left(\frac{Z}{Y}\right)_{sup} = \frac{v}{u_n} g_Z = h_{sup}$$

Reversing the causality that we have in the traditional Kaleckian model, the investment-output ratio determines the saving ratio in the long-run Kaleckian model. This is possible because the saving ratio depends on the autonomous NCCE to output ratio (Z/Y), which is an endogenous variable in the model as a result of the existence of an autonomous NCCE component in aggregate demand. The equilibrium value of Z/Y is:

$$(42)\ \left(\frac{Z}{Y}\right)_{sup} = s_K\pi - h_{sup} = s_K\pi - \frac{v}{u_n} g_Z$$

Note also that $(Z/Y)_{sup}$ is the reciprocal of the long-run equilibrium value of the Supermultiplier, as can be seen from the equation for the determination of the long-run equilibrium level of output given by:

$$(43)\ Y_{sup} = \left(\frac{1}{s_K\pi - \frac{v}{u_n} g_Z}\right) Z$$

The term within the parenthesis is the equilibrium value of the supermultiplier, which is an endogenous variable in the long-run Kaleckian model. The equilibrium value of the supermultiplier is positively related to the growth rate of autonomous NCCE, which implies that $(Z/Y)_{sup}$ is negatively related to the latter variable. Thus, when g_Z increases (decreases), the share of capacity creating expenditures (i.e. the investment-output ratio) increases (decreases). Concomitantly, the latter change leads to an increase (a decrease) in the equilibrium value of the supermultiplier, to a decrease (an increase) in the ratio of autonomous NCCE to output (i.e., $(Z/Y)_{sup}$) and to an increase (a decrease) in the saving ratio. These endogenous changes in the composition of aggregate demand between capacity and non-capacity creating expenditures are the key adjustment mechanism in the long-run Kaleckian model. In fact, the latter adjustment mechanism allows the reconciliation of demand (NCCE) led growth, exogenous income distribution and the convergence towards normal capacity utilization in the long-run model. [23]

One last remark concerns the meaning of the stability condition of what we called the Supermultiplier equilibrium in the long-run model. In traditional Kaleckian models, the usual stability condition is the Keynesian stability one. As we saw in our exposition of a basic version of the latter models, when such a stability condition is met the marginal propensity to spend has value lower than 1 (i.e., we have $1 - s_K\pi + \beta v = c + \beta v < 1$). Thus, the economic meaning of the stability condition is well established in the case of the traditional Kaleckian model and in the case of the analysis of the short-run equilibrium of the Kaleckian models analyzed in this paper. However, things change when we refer to the stability condition of the Supermultiplier equilibrium. In this case, the discussion of the

[23] In essence, this is the distinctive characteristic of the theoretical closure provided by the SM-model (Serrano, 1995b; Serrano & Freitas, 2017).

stability condition usually refers to the necessity of "tackling" and/or "taming" the Harrodian instability principle and the adjustment mechanisms of the type present in equation (10) are called "Harrodian mechanisms". We believe that the discussion of the stability condition of the Supermultiplier equilibrium in these terms is not appropriate.

As in the case of the traditional Kaleckian model modified by the introduction of an investment function based on the capital stock adjustment principle (a type of accelerator investment function), the Harrodian fundamental instability hypothesis follows from the *combination* within the same model of the capital stock adjustment principle and an exogenously determined saving ratio (Serrano, 1995b). The capital stock adjustment principle (the accelerator) cannot be blamed alone for the instability result. In general terms, the problem is that the adjustment of the actual rate of capacity utilization towards its normal level requires appropriate changes in the investment-output ratio. Nevertheless, since the saving ratio cannot change in line with it, the functioning of the capital stock adjustment leads to instability of the normal capacity utilization equilibrium in Harrod's growth model, as well as in the modified Kaleckian model.

Note, however, that the existence of an autonomous NCCE and, consequently, the endogenous determination of the saving ratio characterizes the Supermultiplier equilibrium of the long-run model. Hence, the stability analysis based on the Harrodian instability principle cannot be applied in this specific context. The analysis of the stability of the Supermultiplier equilibrium should be based on a Keynesian/Kaleckian notion of stability. In fact, the stability condition expressed by inequality (40) above says that for the Supermultiplier equilibrium to be locally stable the marginal propensity to spend must have a value strictly lower than one in its neighborhood. Therefore, we argue that the stability condition for the Supermultiplier equilibrium should be interpreted as a generalization of the usual Keynesian stability condition. The stability analysis of such an equilibrium requires the generalization mentioned to be able to deal with the out-of-equilibrium variability of investment-output ratio that is necessary for bringing the actual rate of capacity utilization in line with its normal level.[24]

5. Concluding remarks

We presented a basic version of traditional Kaleckian growth models in order to argue that these models have three important limitations from the point of view of the SM-model: (i) they cannot generate a tendency towards normal capacity utilization; (ii) they do not imply the existence of a positive relationship between the rate of growth of output and the investment-output ratio; and (iii) they fail to give a growth-determining role to NCCEs in the interpretation of real demand-led growth processes. In the sequence, we introduced an investment function based on the capital stock adjustment principle in the place of the traditional Kaleckian one. We showed that the modified model has a Harrodian equilibrium, which is unstable in a strong sense. We argued that the capital stock adjustment principle was not responsible for such instability result. In fact, the latter follows from the *combination* of such a principle with a basic assumption shared by the traditional Kaleckian and the Harrod model according to which an exogenous saving ratio determines the investment to output ratio (i.e., the share

[24] The third term on the left hand side of condition (40) deals with the out of equilibrium changes in the investment to output ratio. Clearly, it depends on the intensity of the process of adjustment of productive capacity to demand governed by the parameter λ of the investment function.

of capacity creating expenditures on output). Therefore, the modified Kaleckian model cannot overcome any of the limitations listed above. In trying to overcome the first of these limitations, among other objectives, Allain (2015), Lavoie (2016) and Dutt (2016) developed two new Kaleckian growth models: the medium-run model and long-run model. We critically analyzed both models.

Our investigation of the medium-run model showed that it has two equilibria, the traditional Kaleckian equilibrium and a kind of hybrid equilibrium. We showed that the traditional Kaleckian (hybrid) equilibrium is stable depending on whether the growth rate of autonomous NCCE is smaller (greater) than the equilibrium rate of growth of the traditional Kaleckian model. We also noted that when the hybrid equilibrium prevails, the rate of growth of the autonomous NCCE determines the rate of growth of output and, hence, NCCEs have a growth-determining role in the medium-run model. Nonetheless, this is not true for the traditional Kaleckian equilibrium of the model, which displays a pattern of economic growth led by investment. Next, we observed that the medium-run model is not able to produce a tendency towards normal capacity utilization and to predict a positive relationship between the investment to output ratio and the trend rate of growth of output. Indeed, in the case of the hybrid equilibrium, we showed that the model predicts the existence of a negative relationship between these two variables. Therefore, the medium-run model cannot completely overcome all three limitations pointed out above.

Moreover, we argued that the medium-run model is not a plausible representation of reality when we think of it as an end in itself and not as an intermediate step towards the long-run model. In fact, we showed that the convergence towards the traditional Kaleckian equilibrium requires that the influence of the autonomous NCCE diminishes until it becomes nil, while the investment to output ratio continuously increases towards its maximum value given by the marginal propensity to save. We believe that such behavior of the model in the traverse towards the traditional Kaleckian model is not reasonable. The problem is that there is no economically meaningful way to restrict the values of the exogenous variables and parameters of the model to prevent the possibility that the traditional Kaleckian equilibrium prevails over the hybrid one. Thus, we cannot exclude the possibility that the medium-run model may present an implausible economic behavior. We suggested that this difficulty is due to the fact that the medium-run model suffers from a “double identity problem”. Our analysis showed that the latter problem follows from the presence, within the same model, of two sources of autonomous aggregate demand, one related to a capacity creating expenditure (i.e., autonomous investment) and the other related to a NCCE (i.e., autonomous consumption), and that it disappears once we remove the autonomous capacity-creating component from the model.

Concerning our analysis of the long-run model, the main results obtained are the following. First, we showed that the model has two equilibria. One of them is a Harrodian equilibrium, while the other is what we called a Supermultiplier equilibrium. The former equilibrium occurs when the ratio of autonomous NCCE to output has a zero value. Our stability analysis showed the Harrodian equilibrium is unstable in a strong sense, meaning that it depends only on the sign (and not on the magnitude) of the parameter responsible for the capacity adjustment process. On the other hand, we showed that, provided the stability condition of the Supermultiplier equilibrium is met, the long-run model can overcome all three limitations of the traditional Kaleckian model. Moreover, we argued that this latter achievement follows from the *combination*, within the model, of an autonomous NCCE component in aggregate demand and an investment function based on the capital stock adjustment principle.

Next, we tried to clarify the important role of the existence of an autonomous NCCE in the long-run model. In this connection, we showed that, even when one considers income distribution to be given from outside the model, the presence of autonomous NCCE in the long-run model allows the endogenous determination of the saving ratio. We remarked that the determination of the saving ratio within the long-run model involves the endogenous determination of the ratio of autonomous NCCE to output and of the equilibrium value of the Supermultiplier. Such an endogeneity of the saving ratio allows the movements of the investment-output ratio governed by the capital stock adjustment principle to bring the actual rate of capacity utilization in line with its normal level. More importantly, the adjustment mechanism based on the existence of an autonomous NCCE component in aggregate demand allows the convergence towards normal capacity utilization to be reconciled with the prevalence of a pattern of demand (NCCE) led growth and with an exogenous determination of income distribution in the long-run model.

We concluded our assessment of the long-run model by discussing the meaning of the stability condition of the Supermultiplier equilibrium of the long-run model. We argued that Harrodian instability principle is only suitable for the stability analysis of an equilibrium whose determination involves the combination of the capital stock adjustment principle and an exogenously determined saving ratio (i.e., the share of capacity-creating expenditure). Since the determination of the Supermultiplier equilibrium involves the functioning of the capital stock adjustment principle combined with the endogenous determination of the saving ratio, we suggested that an analysis based on the Harrodian instability principle cannot be properly used in the investigation of the stability condition of such equilibrium. Instead, we argued that such an investigation should be based on a Keynesian/Kaleckian notion of stability. Therefore, we suggested that the stability condition of the Supermultplier equilibrium should be interpreted as a generalized Keynesian stability condition, which implies the existence of a limit for the value of the marginal propensity to spend. The generalization of the Keynesian stability condition is necessary to be able to deal with the out of equilibrium variability of the investment-output ratio required for the operation of the capital stock adjustment principle in the long-run model.

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